University of Pennsylvania

The Wharton School

Department of Operations and Information Management

Mathematical Modeling and its Application in Finance
OPIM 653 - Spring 2010

Monday & Wednesday, 3:00pm - 6:00pm
January 20 – March 3, 2010
Room: 350 JMHH
Office Hours: Mon. & Wed 1:30-3:00pm or by appointment

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Quantitative methods have become fundamental tools in the analysis and planning of financial operations. There are many reasons for this development: the emergence of a whole range of new and complex financial instruments, innovations in securitization, the volatility of fixed-income markets since interest rate deregulation, the increased globalization of the financial markets, the proliferation of information technology, and so on.

In this course, models for hedging, asset allocation, and multi-period portfolio planning are developed, implemented, and tested. In addition, pricing models for options, bonds, mortgage-backed securities, and swaps are discussed. The models typically require the tools of statistics, optimization, and/or simulation, and they are implemented in spreadsheets or a high-level modeling environment, MATLAB. A student version of Matlab is available at the University Bookstore; details will be discussed in class.

Optional readings in the course bulkpack will focus on the applications and will include recent publications by investment and commercial banks.

The grade for the course will be based on homework assignments and a take-home final exam. The homework will count for 60% of the final grade. There will be about six-seven homework assignments during the term. The final exam will count for 40% of the final grade. Students may work on the homeworks in groups of three or less. Details of the final exam will be discussed later in the term.

This course is quantitative and will require extensive computer use. The course is intended for students who have a strong interest in finance. Prospective students of this course should be comfortable with quantitative methods, such as basic statistics and the methodologies (mathematical programming and simulation) taught in OPIM 621 (Decision Models and Uncertainty).

Course Syllabus

Tentative Schedule

Lecture 0,
>   ***Mathematical Review***. Review of basic probability, statistics, and regression. Financial data analysis. time series analysis, volatility measurement.
>
>   *Readings*: BP sections on Background material and Data Analysis

1   ***Course Overview & Hedging***.
>   The course begins with an introduction to hedging. A hedged portfolio is one that is insulated from market forces. In this section, we discuss some basic concepts in hedging. The issue of hedging will be addressed later in the course when pricing models are developed for specific securities.
>
>   *Topics*: Formulation of the hedging problem, Regression hedging. Examples and applications; in-sample versus out-of-sample performance.
>
>   *Readings*: BP section on Hedging and Value at Risk (VAR).

2-6   ***Numerical Option Pricing – Overview***
>   We first give some brief background on option pricing theory and then present the main numerical methods for pricing complex options – the binomial method and Monte-Carlo simulation. We start with simple European calls and puts, then extend to American and exotic options. Exotics include options with path-dependent payoffs and options on multiple underlying assets.

2   ***Numerical Option Pricing I – Option Pricing Theory***
>   We first give some brief background on option pricing theory and then present the binomial method.
>
>   *Topics:* Review of the Black&Scholes Model; Geometric Brownian process and the lognormal distribution of stock prices; the discrete approximation (CRR model).
>
>   *Readings*: BP section on Numerical Option Pricing.

3-5   ***Numerical Option Pricing II – The Binomial Method***.
>   We start with the binomial pricing method for simple European calls and puts, and then extend it to American options. We continue with binomial pricing methods for exotic options; Exotics include options with path-dependent payoffs and options on multiple underlying assets (rainbow options).
>
>   *Topics:* European and American options. Path independent and path dependent securities: caps, barrier and lookback options. Options on multiple assets.
>
>   *Readings*: BP section on Numerical Option Pricing.

5-6   ***Numerical Option Pricing III – Monte Carlo Simulation***.
>   We discuss Monte Carlo Simulation. Pricing path independent and dependent options.

Variance Reduction Techniques.

Topics: Random number generator; Pricing path independent and dependent securities by simulation; Variance reduction techniques: antithetic and control variate techniques, stratified sampling and Latin Hypercube.

Readings: BP section on Numerical Option Pricing.

Option Pricing IV – Back to the Real World
We apply the B&S models to traded securities and observe the volatility smile.

Topics: Historical simulation, stress test, stochastic volatility, jump diffusion process.

7   Delta Hedging
Black&Scholes meet the real world: We challenge the model's assumptions and partially investigate (via simulation) why observed market prices differ from theoretical values......

Topics: Static hedging versus dynamic hedging, Naked/Covered positions; Stop-Loss strategy.

Structured Option Portfolios
Finally, we investigate the problem of forming a portfolio of short-term options to create a synthetic long-term option. This approach is called structured option portfolio and is motivated by possible difficulties with a standard replication approach for a long-term option.

Topics: Application to long-term option replication. Portfolio option replication.

Readings: BP section on Structured Option Portfolio

8   Portfolio Optimization
In this part of the course, the risk-reward tradeoff is explored for a portfolio with multiple securities. First, we study the standard mean-variance quadratic programming model, variations of the model based on alternative definition of risk, and several applications and extensions.

Topics: ADR and variance measures of risk. Linear and quadratic programming methods for portfolio management and asset allocation. The indexation problem and multiple linear regression. Surplus optimization. Mean-variance analysis with transaction costs. Testing and implementation issues.

Readings: BP section on Portfolio Optimization.

9   Multiperiod Portfolio Analysis.
Finally, we address the multiperiod investment problem and the problem of choosing among efficient portfolios.

Topics: Multiperiod portfolio models. Kelly's criterion. Portfolio insurance strategies. Review of the lognormal distribution.

Readings: BP section on Multiperiod Portfolio Analysis.

10 *Bond Analytics*.
We give a brief background on bond mathematics. We study the pricing of U.S. Treasury bonds and introduce some of the taxonomy used for fixed-income securities such as yield and duration.

Topics: Review of discounting, present value and yield. Duration and convexity measures. Immunization and hedging applications. The discount factor and the spot yield curve.

Readings: BP section on Bond Analytics.

11 *Yield Curve Fluctuations*.
We investigate changes in the Treasury yield curve and its implication for bond portfolio management.

Topics: Bond spreading and hedging. Spot rates, forward rates, term structure estimation. Modes of fluctuation of the yield curve - a principal components analysis. Applications to bond pricing. Volatility of interest rates.

Readings: BP section on Yield Curve Analysis.

12-13 *Interest Rate Models and Pricing Interest Rate Sensitive Securities*.
Finally, using a simple interest-rate model, we look at pricing of bonds and callable bonds, swaps and swaptions, caps and floors, and mortgage-backed securities.

Topics: Interest rate tree and the yield curve. Interest rate models: Ho-Lee, Black-Derman-Toy, Hull-White, and Heath-Jarrow-Morton. Callable bond pricing. Interest rate swaps and swaption pricing. Introduction to the MBS market. Prepayment modeling. MBS pricing models. Stripped MBS.

Readings: BP section on Interest Rate Models.

Course Readings
Most of the reading for the course will be in the form of lecture handouts that will be distributed in class and material from the course bulkpack.

Course Syllabus

Course-Related Books

"Industry" Books

The following books are widely read on Wall Street. Some books are edited compilations of research reports from the major investment banks.

Against the Gods: The Remarkable Story of Risk, Peter Bernstein, Wiley, New York, 1998.

Asset & Liability Management: A Synthesis New Methodologies, Risk Publications/ Kamakura, London, 1998.

Black-Scholes and Beyond: Option Pricing Models, Chriss Neil, Irwin Professional Publishing, Burr Ridge, Illinois, 1997.

Bond Markets, Analysis and Strategies, 6rd edition, Fabozzi and Fabozzi, Prentice Hall, Englewood Cliffs, NJ, 2006.

Capital Ideas: the Improbable Origins of Modern Wall Street, Peter Bernstein, Wiley, 2nd edition, New York, 2005.

Corporate Hedging in Theory and Practice: Lessons from Metallgesellschaft, Eds. C. Culp and M.H. Miller, Risk Publications, 1999.

Credit Derivatives: Application for Risk Management, Investment, and Portfolio Optimisation, Risk Publications, London, 1998.

Derivative Credit Risk, RISK Publications/Renaissance Software, London, 1995.

Derivatives Pricing: The Classic Collection, Peter Carr (Editor), Risk Books, 2004.

Derivatives Trading and Option Pricing, Nicholas Dunbar (Editor), Risk Books, 2005.

Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation, Richard O. Michaud, HBS Press, 1998.

Financial Futures Markets: Structure, Pricing and Practice, John Merrick, Jr., Harper and Row, New York, 1990.

Fixed Income Analytics: State-of-the-Art Debt Analysis and Valuation Modeling, Ravi Dattatreya, ed., Probus Publishing, Chicago, 1991.

Fixed-Income Portfolio Strategies, Frank Fabozzi, ed., Probus Publishing Company, Chicago, 1989.

Fooled by Randomness: the Hidden Role of Chance in Life and in the Markets, Nassim Nicholas Taleb, Random House, 2nd edition, 2005.

From Black-Scholes to Black Holes, RISK Publications/FINEX, London, 1992.

Handbook of Mortgage Backed Securities, 5th edition, F. Fabozzi McGraw-Hill, 2001

Hedging with Trees: *Advances in Pricing and Risk Management Derivatives*, Eds. M. Broadie and P. Glasserman, Risk Publications, London, 1998.

Inventing Money: The Story of Long-Term Capital Management and the Legends behind It, Nicholas Dunbar, John Wiley & Sons, 2001

Liar's Poker; Michael Lewis, Norton, New York 1989.

Monte Carlo: Methodologies and Applications for Pricing and Risk Management, Ed. Bruno Dupire, Risk Publications, London, 1998.

My Life as a Quant: Reflections on Physics and Finance, Emanuel Derman, Wiley, 2004

Over the Rainbow, RISK Publications/Fuji, London, 1996,

Rubinstien on Derivatives, Mark Rubinstein, Risk Publications, London, 1999.

Stocks for the Long Run: the Definitive Guide to Financial Market Returns and Long-Term Investment Strategies, Jeremy Siegel, McGraw-Hill; 3rd edition, 2002.

The Black Swan: The Impact of the Highly Improbable, Nassim Nicholas Taleb, Random House, 2007

The Complete Guide to Option Pricing Formulas, Haug Espen Gaader, McGraw-Hill, 2nd edition, 2006.

The Concepts and Practice of Mathematical Finance, Mark S. Joshi, Cambridge University Press, 2003

Fortune's Formula: The Untold Story of the Scientific Betting System that Beat the Casinos and Wall Street, William Poundstone, Hill and Wang, 2006

The Handbook of Exotic Options: Instruments, Analysis, and Applications, ed.: Israel Nelken, Irwin, Chicago, 1995.

The Handbook of Fixed Income Options, Frank Fabozzi, ed., Irwin Professional Publishing, Burr Ridge, Illinois, 1996.

The Handbook of Fixed Income Securities, 5th edition, Frank Fabozzi, ed., Irwin Professional Publishing, Burr Ridge, Illinois, 1997.

The Winner's Curse: Paradoxes and Anomalies of Economic Life, Richard Thaler, Princeton University Press, Princeton, NJ, 1994.

Trading and Investing in Bond Options, M. Anthony Wong, John Wiley, New York, 1991.

Volatility and Correlation: the Perfect Hedger and the Fox, Riccardo Rebonato, John Wiley & Sons; 2nd edition, 2004.

Volatility: New Estimation Techniques for Pricing Derivatives, ed. Robert Jarrow, Risk Publications, London, 1998.

When Genius Failed: The Rise and Fall of Long-Term Capital Management, Roger Lowenstein, Random House, 2000.

Journals

Many articles relevant to this course appear in finance journals. Some of the journals that are pitched more towards practitioners include: *Journal of Derivatives, Journal of Fixed Income, Journal of Portfolio Management, Journal of Computational Finance Financial Analysts Journal, and Risk Magazine*. Some of the academic journals include: *Journal of Finance, Journal of Financial and Quantitative Analysis, Journal of Futures Markets, and Review of Financial Studies*.

Finance Textbooks

Cox and Rubinstein, *Options Markets*, Prentice Hall, Englewood Cliffs, NJ, 1985.

Dixit and Pindyck, *Investment Under Uncertainty,* Princeton University Press, Princeton, NJ, 1994.

Elton and Gruber, *Modern Portfolio Theory and Investment Analysis*, 5th edition, Wiley, New

York, 1995.

Garbade Kenneth, *Fixed Income Analytics,* MIT Press, Cambridge, MA, 1997.

Hull John, *Options, Futures, and other Derivatives*, 7th edition, Prentice Hall, NJ, 2008.

Jarrow and Turnbull, *Derivative Securities*, 2nd edition, South-Western College Pub., 1999.

Luenberger David, *Investment Science*, Oxford University Press, 1998.

Markowitz, *Mean-Variance Analysis in Portfolio Choice and Capital Markers*, Blackwell, New York. 1987.

Rebonato Riccardo, *Interest-Rate Option Models: Understanding, Analysing and Using Models for Exotic Interest-Rate Options*, 2nd edition, Wiley, NY, 1998.

Rodriguez and Carter, *International Financial Management*, 3rd edition, Prentice Hall, Englewood Cliffs, NJ, 1984.

Shreve Steven, *Stochastic Calculus Models for Finance: Continuous Time Models*, Springer 2005.

Sundaresan, Suresh, *Fixed Income Markets and their Derivatives,* 2nd edition, South-Western, Cincinnati (2002).

Tuckman, Bruce, *Fixed Income Securities: Tools for Today's Markets*, 2nd edition, Wiley, NY, 2002.

Wilmott Paul, *Paul Wilmott on Quantitative Finance,* 2nd edition, John Wiley & Sons, 2006

Statistics Textbooks

Newbold Paul, *Statistics for Business and Economics*, Prentice Hall, 1989.

Smith Gary, *Statistical Reasoning*, Allyn and Bacon, Boston, 1985.

Time Series and Multivariate Statistics Textbooks

Johnson and Wichern, *Applied Multivariate Statistical Analysis*, Prentice Hall, Englewood Cliffs, NJ, 1982.

Maxwell, *Multivariate Analysis in Behavioural Research*, Wiley, New York, 1977.

Pankratz, *Forecasting with Univariate Box-Jenkins: Concepts and Cases*, Wiley, New York, 1983.

Pindyck and Rubinfeld, *Econometric Models and Economic Forecasts*, McGraw Hill, New York, 1976.

Taylor, *Modeling Financial Time Series*, Wiley, New York, 1986.

Operations Research Textbooks

Eppen, Gould and Schmidt, *Introductory Management Science*, 4th edition, Prentice Hall, 1993.

Winston, *Operations Research*, 3rd edition, Duxbury Press, Belmont, CA, 1994.

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